AN EXPERIMENT IN PARTITIONING SECOND-GRADE MATHEMATICS CLASSES INTO SMALLER SECTIONS OF COMPARABLE ABILITY

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THESIS

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ABSTRACT

The mathematics program of an elementary school was changed so that the classes were partitioned into smaller sections of comparable ability. This change reduced the pupil-teacher ratio and the length of the instruction period. The proposal produced an unanswered question: Will there be an increased achievement gain during the school year which can be attributed to the program change? Two schools served as control groups for the experiment. The California Achievement Test, 1970 Edition, was used in a pretest-posttest design to measure achievement levels before and after the elapsed time of the experiment. The question was answered in the affirmative through the use of various statistical techniques: randomized matched subjects design, analysis of covariance and the non-parametric Mann-Whitney U test. The several techniques were applied since no single standard practice was available for this problem.

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The author is deeply indebted to Douglas J. McRae of CTB/McGraw-Hill for his knowledge, assistance, and the loan of reference material. Specific indebtedness was incurred for his School Characteristics Questionnaire and his cluster analysis computer program which he developed as part of his doctoral dissertation while at the University of North Carolina, Chapel Hill. The author is further indebted to CTB/McGraw-Hill for supplying their California Achievement Test, 1970 Edition, and for permission to reproduce portions of the standardization data for the California Achievement Test, 1970 Edition.

I. INTRODUCTION

On September 15, 1970, the author attended a PTA meeting at his child's school. The primary purpose of the meeting was to discuss a proposal for a change in the mathematics instruction at the second grade level and to obtain the approval of the parents involved.

The proposal was primarily directed at the large pupil-teacher ratios, approximately 30 to 1, existing in the five second grade classrooms. Essentially, the pupils in the second grade would be divided into ten sections (vice five classes), with five of the sections being taught in the morning and five in the afternoon. The same five classroom teachers would be teaching both sessions. The pupil teacher ratio would be virtually halved by the proposed change. Also the smaller sections were to be composed of pupils of comparable ability. The parents were unaware of this aspect of the proposal.

A scheduling problem was inherent in the proposal due to the morning and afternoon sessions and the limited number of teaching hours available during the day. This problem was solved by having the second graders arrive and depart the school at times different from the normal school routine and by reducing the normal 40 minute math period to 30 minutes in the morning session and to 35 minutes in the afternoon session. These scheduling changes allowed the teachers to remain within their alloted teaching hours.

The main point of concern to the parents centered around the possible inconvenience caused by the different arrival and departure times of the second graders involved. In the discussion that followed it was implicitly assumed that the achievement level of the pupils would be higher under the proposed change; this assumption was never questioned. The primary reason for the assumption was the reduced pupil-teacher ratio made possible by the increased number of sections. The reduction of the math periods by ten minutes in the morning session and five minutes in the afternoon session was not felt to be significant by comparison.

The proposal was adapted and implemented the following week without the question of increased achievement being considered. The objective of this thesis, then, is to determine if there was a significant increase in mathematical ability of the second graders involved which could be attributed to the change in mathematics instruction.

Measuring and evaluating growth objectively is a classical problem in psychometrics. It has been debated extensively in the literature by Lord [Ref. 1, 2, and 3], McNemar [Ref. 4], Cronbach and Furby [Ref. 5] and others. Reference 6 is a collection of papers all dealing with the same problem - change: how to measure and evaluate it. Cronbach and Furby have even gone so far as to suggest that measurement of change should not be done. They do present a model for analyzing individual change which they consider b tter than

other available models, but they recommend that the results be interpreted with caution. The purpose here is neither to add nor subtract from the sometimes analytical, sometimes philosophical, arguments found in the literature but to proceed with models that have been used in the past to measure and evaluate change.

It was probable that mathematical skills would increase over the school year with or without the implementation of the proposal. This created a problem in isolating that portion of achievement, if any, due to the adopted proposal. This and other extraneous factors led to the selection of a control group. Two requirements were considered necessary for the control group. The first was that the pupils be assigned to classes in the normally accepted manner and that classes not be divided into smaller sections. The second requirement was that the pupils in the control group be of comparable ability with those in the experimental group. The first requirement seemed easy enough to satisfy, but the second seemed very difficult without prior knowledge of mathematical ability of both groups.

An objective measuring device was also needed to measure mathematical ability. The most commonly accepted way of measuring knowledge in educational institutions is by the use of tests, be they written, oral, or other. It was decided to give a written pretest and posttest to both the experimental and control groups. It was felt that this pretest-posttest design would alleviate some of the difficulties in meeting the second requirement mentioned above.

Fortunately, the author was referred to CTB/McGraw-Hill, Monterey, California, by his thesis advisor. Douglas J. McRae, of the research department of that firm, took an interest in this project. Upon his urging, the firm agreed to furnish their California Achievement Tests, 1970 Edition, to the schools involved. Consultation with Mr. McRae and the principal of the experimental school also led to the selection of a control group.

It is widely accepted that a positive correlation exists between socio-economic level and academic achievement. It was, then, on the basis of assumed socio-economic level that the experimental and control groups were deemed to be of comparable ability. A questionnaire furnished by Mr. McRae was used by the author to establish the socio-economic levels of the two groups.

The experimental group happened to be in the largest elementary school in the area, so two schools were chosen to form the control group. This allowed the numerical population of the experimental and control groups to be of the same magnitude and the results to have a broader base. The principals of the two schools tenatively selected as the control group were contacted and both were eager to cooperate in the experiment. The experimental school was labeled school A and the control schools were labeled B and C.

II. PUPIL-SCHOOL BACKGROUND

A School Characteristics Questionnaire [Appendix A], furnished by Douglas J. McRae of CTB/McGraw-Hill, was given to the principal of each school to complete. The questionnaire requested information on a variety of pupil, staff, and physical plant characteristics. The schools were all considered to be located in residential suburbs. The pertinent results of the questionnaire are presented in Table I. The number in parenthesis refers to the questionnaire item.

QUESTION AREA \ SCHOOL (ENROLLMENT)		A(900)	B(400)	C(600)
STUDENT DEMOGRAPHIC:				
Mobility	(2)	35%	10%	15%
PTA Attendance	(4)	20%	20%	10%
Employed Mothers	(6)	5%	25%	40%
Per Cent White	(10)	98%	95%	95%
Kindergarten	(13)	75%	100%	100%
One Parent	(14)	.2%	5%	10%
English Second	(15)	.4%	1%	2%
Professional	(16)	75%	50%	45%
White Collar	(16)	15%	30%	37%
Skilled	(16)	10%	15%	15%
Unskilled	(16)	0%	5%	3%
PHYSICAL PLANT AND ADMINISTRATIVE:				
Plant Age	(3)	17 yrs	5 yrs	20 yrs
New Programs	(8)	2-5 mos	2-6 mos	2-6 mos
Library	(17)	7650	-	9000
STAFF CHARACTERISTICS:				
Principal's Salary	(11)	$17,145	$17,500	$16,500
Average Start Salary	(12)	7,500	7,500	7,500
Average Salary	(19)	9,850	-	10,000
Average Experience	(18)	7 years	7½ years	9 years

TABLE I. RESULTS OF SCHOOL CHARACTERISTICS QUESTIONNAIRE.

It is apparent that the schools differ in several areas (e.g., mobility, employed mothers, and the number of one parent homes). There is an apparent positive correlation between the per cent of working mothers and the per cent of homes with one parent. The experimental school had a lower percentage of pupils who had attended kindergarten. This agreed with the higher mobility figures for that school, as it is plausible that a large percentage of the pupils came from a school district outside of California, where kindergarten is not required. Two of the schools (A and C) have considerably older physical plants, but all three schools maintain a clean, neat appearance conducive to learning.

In other areas the schools were very similar. They were homogeneous racially; at least 95 per cent of the pupils from each school were white. They were all members of the same school district; hence they had the same minimum math program [Appendix B]. The average experience of the teaching staff was between seven and nine years. At least 80 per cent of the pupil's parents were in white-collar or professional occupations. From this it can be assumed that the socio-economic level of the three schools was approximately the same, partially confirming the basis on which the control schools were chosen.

III. ASSIGNMENT TO CLASSES AND INSTRUCTIONAL LEVELS

The school district had set forth a statement of minimum goals and skills to be achieved by the second grade in mathematics [Appendix B]. All three schools involved in the experiment used Modern School Mathematics - Structure and Use, published by the California State Department of Education, as the basic text. This text adequately covers the minimum requirements and exceeds them, continuing into multiplication. More advanced texts are available for the gifted child, who is allowed to proceed at an accelerated pace. The advanced texts extend the concepts covered in the basic text and introduce the pupil to division.

In the experimental group, pupils were initially assigned to one of ten mathematics sections on the basis of reading performance and a departmental math test. Some assignments were changed at two different times during the school year. The first shift occurred after the pretest was given. The pupils who scored way above the average for their section were shifted to an appropriate section. No pupils were transferred to a lower section on the basis of the pretest.

The second assignment change occurred during the middle of the school year. Several pupils transferred to and from the school. Persons in authority felt that the new pupils should not be placed in advance sections; not enough information was available for basing the higher assignment.

Hence new students were placed in the lower sections. The better pupils in the lower sections were shifted to advanced sections to keep the pupil-teacher ratios comparable. It was felt that by shifting pupils in this manner no one would be in a section in which the material presented was beyond his capability. Unfortunately, the records as to which pupils were shifted are not complete. This led to a difficulty in interpreting the data, and the question is addressed later in the thesis.

Five of the sections were taught during the morning and five during the afternoon. This division of the five second grade classes into ten sections automatically reduced the pupil-teacher ratio and, as noted previously, grouped pupils of comparable ability in the same section. This sectional assignment by ability allowed the teacher to present instruction at the average achievement level of their section rather than the average level of the second grade taken as a whole.

Basically, there were five levels of instruction at the experimental school. Level I was taught at an advanced level to include multiplication and division. Levels II and III were taught at progressively lower levels. Level IV was considered to be composed of pupils who were capable of learning at the normal second grade level. This group was introduced to multiplication near the end of the school year. Level V consisted of the under-achievers. They were taught at the second grade level but with a more repetitious learning process and extensive drills in fundamentals.

In the control schools, levels were not used. Children were assigned to class more or less at random; there was no distinction made between slow and fast achievers. During the school year the level of instruction was based upon the average level of achievement within the class. The teachers gave special assistance to those children who were lagging and to those who were capable of progressing more rapidly. This assistance allowed the pupils in the control schools to have essentially the same range of instruction as that of the pupils in the experiment group.

The division of the five classrooms in the experimental school into ten sections formed the basis for the experiment. These sections allowed for five different levels of instruction and a reduced pupil-teacher ratio of 15 to 1 for levels I, II, III, and IV and 10 to 1 for level V. This is to be contrasted with the control group schools where pupils were assigned more or less at random to a class, one general level of instruction prevailed, and the pupil-teacher ratios were approximately 25 to 1 and 30 to 1 in schools B and C respectively. Further, the pupils in the experimental school were being taught five or ten minutes less per day[1] than the pupils in the control schools.

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[1] Ten minutes for the morning classes and five minutes for the afternoon classes.

IV. THE CALIFORNIA ACHIEVEMENT TEST

The experimental and control groups participating in the experiment and the experiment itself have been described. Information was presented which indicates that the pupils involved were of the same socio-economic level. Presumably, then, the pupils of the different schools were, on the average, at the same level of learning at the start of their second grade year. It is recognized, however, that the maturity level of second graders can range from kindergarten to the third or fourth grade, i.e., it was not assumed that all pupils had reached the same level of academic achievement.

Considering the similarity of starting levels, then, one would expect the same magnitude of achievement in mathematical skills during the second grade school year for both groups, if there were no real difference between the math programs of the experimental and control groups.

As mentioned previously, the California Achievement Test, 1970 Edition, was the measuring device used to measure the achievement level at the beginning and end of the experiment. The test is designed for the measurement, evaluation and analysis of school achievement in the three basic skill areas: reading, mathematics, and language.

These three skill areas are measured separately. The math section of the CAT measures [Ref. 7]:

"-the ability to understand the meaning of the material presented,

-the performance of the student in applying rules, facts, concepts, conventions, and principals of problem solving in the basic curricular material, and
-the level of performance of the student in using the tools of mathematics in progressively more difficult situations."

How well does the CAT measure mathematical achievement? This question is partially answered by Reference 8. Further information, based upon national standardization data, will be available when the <u>Technical Report, California Achievement Test, 1970 Edition</u>, [Ref. 9], is published. The author has had limited access to this material through Mr. McRae, and from all reports the test is well established.

Reference 7 contains tables to assist in transforming raw scores (total number of right responses) into Achievement Development Scale Scores (ADSS). The scale scores were produced by applying the Thrustones absolute scaling procedure to the standardization data [Ref. 8 and 10]. There are five levels of the CAT designed to cover grades one through twelve. The scale scores have the advantage of being articulated across all levels of the CAT, i.e., theoretically a first graders achievement can be compared with that of a student in high school. Further advantages of scale scores over raw scores are that they have interval scale properties and are normally distributed [Ref. 8]. Hence, these scores are advantageous for use in statistical analysis of the measurement of growth, trends, etc.

There are two levels of the CAT which pertain to the second grade. Level 1 is designed to test pupils in the

first and second grade. Level 2 covers the second through fourth grade, thus overlapping level 1 at the second grade. It was decided to give level 1 at the beginning of the school year and level 2 at the end of the school year. The reason two levels were chosen instead of using level 1 twice, was to allow for maximum spread of the scores over the learning period and to avoid practice effects. Interlevel articulation is a correlation measure of how well the different levels measure the same attribute (in this case, mathematical achievement) at the overlap of the levels. The interlevel articulation coefficient for levels 1 and 2, CAT, 1970 Edition, is .78 [Ref. 9].

Level 1 of the CAT was given to all participating schools during the second week of October, 1970. Level 2 was given during the third week of May, 1971, an elapsed time of seven months. The tests were given, monitored and scored by the schools in accordance with Reference 7. The raw scores were collected by the principals of the participating schools and given to the author within a reasonable period of time after each testing period.

Each level of the CAT mathematics section is divided into two parts: concepts and computational ability. The raw scores of each area were kept separate. The author converted the raw scores into scale scores. These scores for all pupils are contained in Appendix C.

V. METHODS OF ANALYSIS

The stated objective of this thesis was to measure the achievement gain in mathematical skills of three different sets of pupils, compare these measures and determine by some means whether any one of these measures is significantly different from the others. As noted in the Introduction, a standard model for answering this question has not been obtained to everyone's satisfaction.

Hence, due to a lack of a totally defensible mathematical model, several statistical techniques were applied to the data. This use of several techniques had the advantage of compensating for weaknesses in different procedures and of developing experience in the various statistical techniques. Naturally the results would be highly correlated and the reinforcing value of a second or third successful procedure is slight (one should not perform multiple tests on a single set of data). Joint probability statements cannot be made, only individual (or marginal) ones. It was still held to be valuable to have the several tests as backups for one another. The three models used in the analysis were the Randomized Matched Subject Design, the Analysis of Covariance, and the non-parametric Mann-Whitney U Test. The reader is asked to judge for himself as to which technique supports the experimental paradigm.

A. RANDOMIZED MATCHED SUBJECT DESIGN [Ref. 11 and 12]

In this test there must be an equal number of pupils in the experimental and control groups. Schools B and C were

combined to form a control group of 113 subjects. School A had 94. Random number tables were used to discard the 19 excess pupils from the control group. After the pretest was given, the pupils were ranked within the experimental and control groups on the basis of their pretest score. The two groups were then paired by placing the pupil scoring highest in the experimental group with the pupil scoring highest in the control group, etc., until 94 pairings were obtained. After the posttest was given and scored a difference score was obtained on each pair in the following manner:

$$D_i = (Y_{E_i} - Y_{C_i}) - (X_{E_i} - X_{C_i}),$$

where

X_{C_i} = pretest of i^{th} pupil in the control group

Y_{C_i} = posttest of i^{th} pupil in the control group

X_{E_i} = pretest of i^{th} pupil in the experimental group

Y_{E_i} = posttest of i^{th} pupil in the experimental group.

With a difference score thus defined, the question of whether or not the program of the experimental group was better than that of the combined control group can be addressed with the following statistical hypotheses:

$$H_o: \mu = 0$$

$$H_1: \mu > 0$$

where μ is the mean of the difference scores. If during the analysis, H_o, the null hypothesis, is not rejected, this will

imply that no significant difference exists in the programs of the experimental and control groups. It was assumed that the difference scores had a normal distribution with a mean of zero and an unknown variance. Under these assumptions, the Student-t distribution yields the uniformly most powerful test against all one-sided alternative hypotheses. It was used in this thesis to test the null hypothesis presented above.

The randomized matched subjects design has a distinct advantage over a design where the difference between the estimated gains made by the experimental and control groups is hypothesized to be zero. The pairing on the pretest scores introduces a high correlation between the groups on posttest scores. This correlation decreases the estimate of the variance by an amount equal to twice the covariance of the means. Thus the randomized matched subjects design is more sensitive to differences between the two groups than a design where pairing is not done, since the denominator of the critical ratio is reduced.

The primary disadvantage of the randomized matched subjects design is that an implicit assumption is made, i.e., the scores obtained on the protests and posttests are true scores. In reality the obtained score on any test can be thought of as a true score plus error, where the error can be modeled as a normally distributed random variable with zero mean and unknown variance. The difference score for each pupil, as defined, is the difference of two normally

distributed random variables. The sum of these differences has a normal distribution but the variance may change from pupil to pupil. Thus the Student-t procedure will estimate a variance which is actually a composite of each students variance rather than the value common to all. Hence this procedure for comparing growth may be questioned.

B. ANALYSIS OF COVARIANCE [Ref. 13]

The second method used in the analysis of the different mathematics program was the Analysis of Covariance. In the analysis the posttest score is considered to be dependent upon the pretest score, which is called the concomitant variable. The Analysis of Covariance takes advantage of the information furnished by the concomitant variable in a sense that the posttest scores are "corrected" by taking into account the differences between the groups on the concomitant variable. After the posttest scores are "corrected" the procedure reduces to a straight analysis of variance. Thus the results from the analysis of covariance more accurately reflect any differences that are due to the different programs than a straight analysis of variance would on the posttest scores.

The assumptions of the analysis of covariance are:

1. A random sample of size one is drawn from each of N populations.

2. Each of the populations is normal and has the same variance.

3. The population means within each group lie on a straight line and the slope of the line is the same for each group.

Since the concomitant variable is measured with error (i.e., the obtained scores are not necessarily true scores) it must also be assumed that the assumptions hold for all possible values of the concomitant variable.

The model for the analysis of covariance can be written as:

$$y_{ij} = \mu + \beta_j + \gamma\,(x_{ij} - \bar{x}_{..}) + e_{ij} \qquad \begin{array}{l} i=1,2,\ldots,n_j \\ j=1,2,..,r \end{array}$$

$$e_{ij} \sim N(0,\sigma^2)$$

$$\sum_{j=1}^{r} n_j\,\beta_j = 0 \qquad N = \sum_{j=1}^{r} n_j$$

where

r = the number of schools.

n_j = the number of pupils in the j^{th} school.

x_{ij} = pretest of i^{th} pupil in the j^{th} school.

y_{ij} = posttest of i^{th} pupil in the j^{th} school.

β_j = program effect of j^{th} school.

γ = slope of regression line (posttest on pretest).

μ = the average of all r population means.

The objective of the analysis is to choose between the following set of hypotheses:

$$H_0: \ \beta_j = \beta \qquad j=1,2,\ldots,r.$$

$$H_1: \ \text{not all } \beta_j \text{ are the same}$$

The null hypothesis implies that the corrected program effects are all equal.

The robustness of this test is not as well known as that of the straight analysis of variance. Feldt [Ref.14], in a paper comparing several techniques, stated that the number of assumptions required for a valid analysis of covariance renders the technique generally less applicable than other statistical tests used for the same purpose. The failure of the data to meet the assumptions is thought to be more serious in analysis of covariance than in straight analysis of variance (the Student-t in the two sample case) especially with failure to satisfy regression assumptions.

Feldt investigated the precision of a factorial design and of the analysis of covariance. He discovered that for regression correlations of greater than 0.6 analysis of covariance has the greater precision. Since CTB/McGraw-Hill has estimated this correlation (their interlevel of articulation) to be 0.78[2] in their standardization procedures, it was believed that the analysis of covariance in this thesis would yield valid results.

C. MANN-WHITNEY U TEST

A third test was presented for those readers unwilling to accept the assumptions in the analysis of covariance or the validity of the randomized matched subjects design. It

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[2] See the r values in Tables IX and Figure 1 for values obtained during the analysis.

is the non-parametric Mann-Whitney U test. In this test it was again necessary to form the two control schools into one, but it was not necessary to delete the 19 pupils as in the randomized matched subjects design. The achievement gain score (posttest-pretest) was computed for all pupils. Again the thesis in question could be answered by a set of hypotheses

H_o: The achievement gains for the experimental and control groups have the same distribution.

H_1: The achievement gains for the experimental group are stochastically larger than those of the control group.

There are no assumptions made as to the distribution of the achievement gains other than the distributions are continuous and that the observations are completely independent. Siegel [Ref. 15] states that the power efficiency of the Mann-Whitney U test approaches 95.5 per cent for large sample sizes compared to the Student-t. It is, therefore, an excellent alternative to the Student-t, and it does not have the restrictive assumptions associated with the analysis of covariance.

VI. INITIAL RESULTS

The set of hypotheses presented in each of the three different techniques of analysis were all designed to answer the same question: Was the achievement level in mathematics of the pupils in school A larger than the gain normally attributed to the learning process during the second grade? If the null hypothesis was not accepted during the analysis it would be a strong indication that the change in mathematical programs at school A contributed significantly to the gains made by the pupils. If the opposite occurred, it could be concluded that the program had no significant effect.

Achievement Development Scale Scores were used in all statistical analysis and the significance level of all tests was arbitrarily set at the 5 per cent level. This significance level is the probability of a Type I error, i.e., the probability that the null hypothesis is rejected when in reality it is true. The power of the tests, i.e., the probability of failing to reject the null hypothesis when the alternative is true, were not determined.

All three statistical techniques discussed were used on the total test score and each of the subtest scores, i.e., grasp of concepts and computational ability. The basic results are presented in Table II. The table includes the estimates of the mean and standard deviations for each of the three scores. The remaining results in this section apply only to the total scores.

SCHOOL	COMPLETE				CONCEPTS				COMPUTATIONS			
	PRE		POST		PRE		POST		PRE		POST	
	$\bar{X}$	S_x	$\bar{Y}$	S_y	$\bar{X}$	S_x	$\bar{Y}$	S_y	$\bar{X}$	S_x	$\bar{Y}$	S_y
A	280.5	24.0	342.8	50.8	327.	38.4	391.2	49.8	276.9	20.6	331.2	46.2
B	276.4	27.8	313.3	39.6	320.6	37.3	366.2	46.4	273.6	25.2	303.7	35.2
C	279.1	30.4	317.8	35.1	327.7	38.0	362.3	43.2	279.8	28.3	311.6	31.4

TABLE II. ESTIMATES OF THE MEANS AND STANDARD DEVIATIONS.

In the randomized matched subject design, the estimate of the mean of 94 difference scores was 24.4. The sample standard deviation was 48.7. These figures combined to yield $t = 4.86 > 1.99$. Hence the null hypothesis was rejected.

The results of the analysis of covariance are contained in Table III. $F = 13.03 > 3.00$, hence the null hypothesis was rejected. The rejection of the null hypothesis in the analysis of covariance only indicates that all

SOURCE	SS_x	SP_{xy}	SS_y	SS'_y	d.f.	MS'_y
TREATMENT	528.4	3,814.1	37,606.9	30,165.2	2	15,002.6
ERROR	150,788.5	157,041.1	398,519.6	234,966.5	203	1,157.5
TOTAL	151,316.9	160,855.2	436,126.44	265,131.7		

TABLE III. ANALYSIS OF COVARIANCE, TOTAL SCORE.

programs were not the same. A technique exists to determine which program or programs were significantly different. A linear combination of the form

$$L = c_1\beta_A + c_2\beta_B + c_3\beta_C \ , \ \sum_{j=1}^{3} c_j = 0$$

where the c_j's are constants, is called a contrast. Scheffé's method [Ref. 15] was used to obtain confidence intervals for several contrasts and to determine which contrasts were significantly different from zero. β_i, $i = A, B, C$ are the program effects of the different schools. The results are contained in Table IV.

CONTRAST	INTERVAL
$\beta_A - \beta_B$	$10.5 \leq L \leq 39.9$
$\beta_A - \beta_C$	$10.0 \leq L \leq 37.0$
$\beta_B - \beta_C$	$-17.5 \leq L \leq 14.2$
$\beta_A - \frac{1}{2}(\beta_B + \beta_C)$	$11.8 \leq L \leq 35.2$

TABLE IV. CONFIDENCE INTERVAL FOR CONTRASTS, TOTAL SCORE.

It is obvious from Table IV that school A is the reason the null hypothesis was not accepted. The confidence intervals associated with school A do not contain zero and are greater than zero.

The Mann-Whitney U test yielded the same result, i.e., the null hypothesis was rejected. The achievement gain scores yielded a U statistic of 6884.5 (correction for ties was included). This transformed into a normalized variate, $z = 3.667$. The probability of this occurring is $0.0013 < 0.05$.

VII. DISCUSSION OF THE RESULTS AND FURTHER ANALYSIS

All three methods of analysis soundly rejected the null hypothesis. Hence, from the preliminary results, it was concluded that the division of classes into small sections, composed of pupils of comparable ability, significantly increased the achievement gains during the second grade school year.

At this point the author decided to determine if the programs were significantly different in both areas of the California Achievement Test. The analysis of covariance was chosen as the technique for further analysis based on the author's belief that it yielded the most information and that it was broader based since the control schools were not combined. The results of the analysis of the concept scores are presented in Table V and computation scores in Table VII. The confidence intervals for contrasts are contained in Tables VI and VIII, respectively.

SOURCE	SS_x	SP_{xy}	SS_y	SS'_y	d.f.	MS'_y
TREATMENT	1,728.4	2,708.5	38,634.4	35,482.1	2	17,741.0
ERROR	298,927.5	230,133.3	457,982.5	280,811.4	203	1,383.3
TOTAL	300,655.9	232,891.8	496,616.9	316,293.5		

TABLE V. ANALYSIS OF COVARIANCE, CONCEPT SCORE.

For concept scores the statistic $F = 12.83 > 3.00$, hence the null hypothesis was also rejected for concepts, and from Table VI it is clear that school A was the reason.

CONTRAST	INTERVAL
$\beta_A - \beta_B$	$3.8 \leq L \leq 36.1$
$\beta_A - \beta_C$	$14.7 \leq L \leq 44.2$
$\beta_B - \beta_C$	$-7.9 \leq L \leq 26.7$
$\beta_A - \frac{1}{2}(\beta_B + \beta_C)$	$12.0 \leq L \leq 37.5$

TABLE VI. CONFIDENCE INTERVALS FOR CONTRASTS, CONCEPT SCORE.

SOURCE	SS_x	SP_{xy}	SS_y	SS'_y	d.f.	MS'_y
TREATMENT	392.6	3,363.3	29,037.6	23,948.9	2	11,974.4
ERROR	122,137.1	95,356.0	320,972.5	250,525.3	203	1,239.1
TOTAL	122,529.7	98,719.3	359,010.1	274,479.2		

TABLE VII. ANALYSIS OF COVARIANCE, COMPUTATION SCORE.

CONTRAST	INTERVAL
$\beta_A - \beta_B$	$9.8 \leq L \leq 40.1$
$\beta_A - \beta_B$	$4.0 \leq L \leq 32.0$
$\beta_B - \beta_C$	$-23.3 \leq L \leq 9.4$
$\beta_A - \frac{1}{2}(\beta_B + \beta_C)$	$9.4 \leq L \leq 33.6$

TABLE VIII. CONFIDENCE INTERVALS FOR CONTRAST, COMPUTATION SCORE.

From the analysis presented, thus far, it is clear that the program at school A allowed for sizeable achievement gains in all math areas of the CAT. A regression of the posttest scores on the pretest scores, total scores, was done to see how well the analysis of convariance assumptions were met. The results are contained in Table IX. The

slopes of the regression equations were within reason for all of the schools, but the correlation was low for school A[3]. Histograms were made for all three schools on the pretest and posttest total scores. All appeared to be normally distributed except the posttest scores of school A. A bimodal distribution appeared, and further analysis was suggested.

SCHOOL	REGRESSION EQUATION	CORRELATION
A	$Y = 18.27 + 1.16\ X$	0.545
B	$Y = -4.40 + 1.15\ X$	0.808
C	$Y = 75.93 + 0.87\ X$	0.750

TABLE IX. REGRESSION ANALYSIS ON TOTAL SCORES.

An investigation was begun to see if the apparent bimodal distribution of posttest scores of school A was associated with the section assignments. At this time it was discovered that complete details of final section assignments were not available. The final assignments were different from the original because of shifts during the school year. The reason for the shifts were explained in detail in Section III of this thesis. The assistant principal and the only available teacher concerned with the experiment assisted

[3]It is possible, through statistical techniques, to test the slopes for equality and to obtain confidence intervals for the correlation. Equality of slopes was assumed in the Analysis of Covariance and the tests were not done to avoid debate about the significance level of the Analysis of Covariance.

the author in verifying 79 (out of 94) of the final assignments. This information was not considered to be complete enough for further analysis.

A cluster analysis computer program was obtained from Mr. McRae [Ref. 16] to separate the pupils into clusters based on their test scores. The program contained several different methods to accomplish this. Two of the methods were used with identical results. The program separated the pupils of school A into two clusters; basically, the clusters consisted of levels I and II, labeled cluster A; and levels III, IV, and V, labeled cluster B. Since 79 of the final assignments were known, it was easy enough to verify the soundness of the routines. Four errors were detected. Three pupils who were known to be in cluster A were assigned to B and one who was known to be in cluster B was assigned to A. The author considered the number of known errors to be sufficiently small to avoid appreciable discrepancies in further analysis.

Another Analysis of Covariance was run on the total scores. This time four groups were included in the analysis: School A-cluster A, school A-cluster B, school B, and school C. The results are presented in Tables X and XI.

The statistic $F = 99.29 > 2.6$. Hence the null hypothesis was rejected. It is clear from Table XI that the scores from school A-cluster A accounted for the rejection of the null

hypothesis. Cluster A consisted of those pupils assigned to levels I and II where the level of instruction was the most advanced.

SOURCE	SS_x	SP_{xy}	SS_y	SS'_y	d.f.	MS'_y
TREATMENT	10,539.9	48,806.3	239,804.1	157,992.7	3	52,669.2
ERROR	140,777.0	112,048.9	196,322.3	107,139.0	202	530.4
TOTAL	151,316.9	160,855.2	436,126.4	265,131.7		

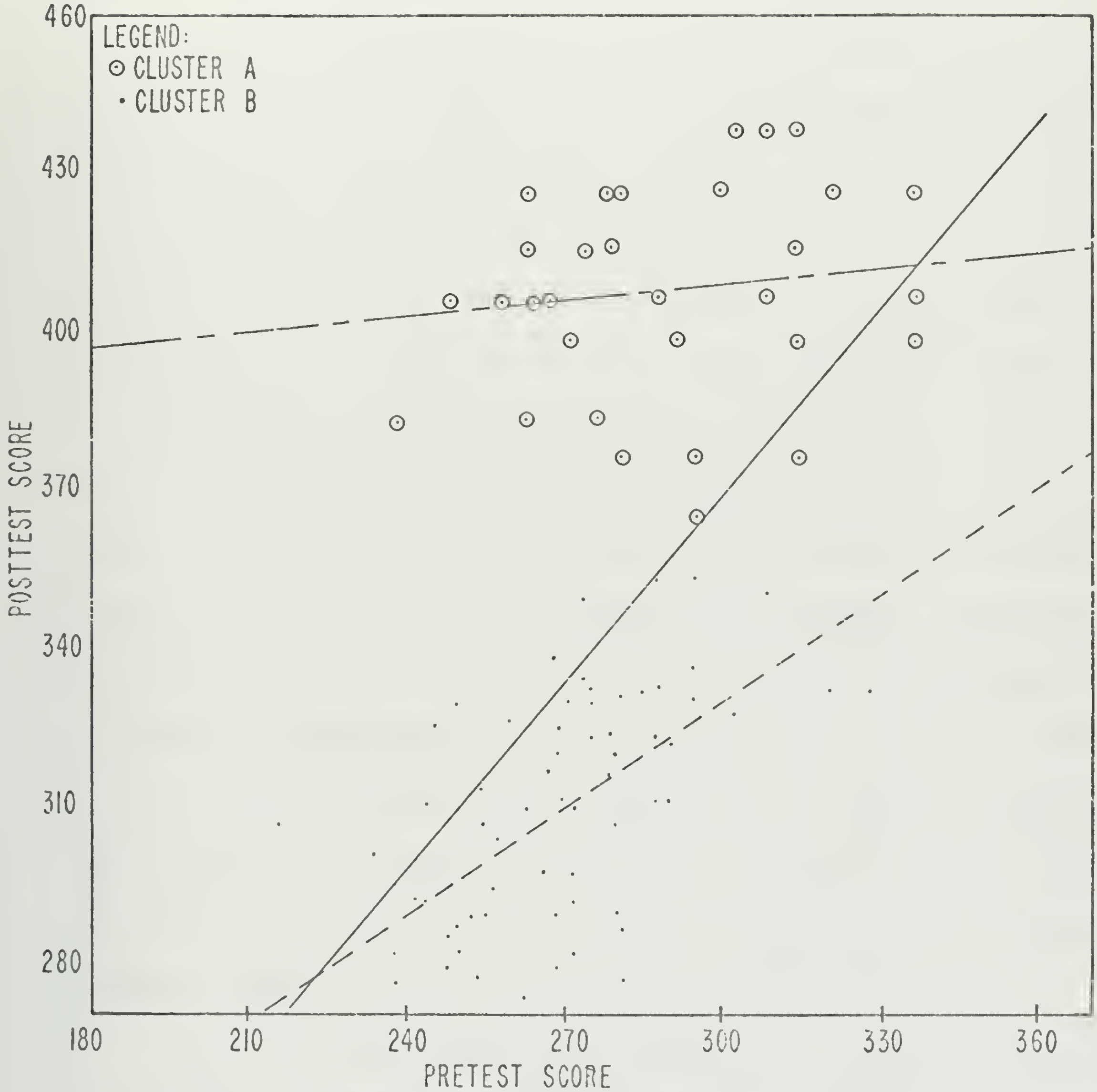
TABLE X. ANALYSIS OF COVARIANCE, TOTAL SCORE. SCHOOL A SEPARATED INTO TWO CLUSTERS.

CONTRAST	INTERVAL
$\beta_{AB}-\beta_B$	$-13.5 \leq L \leq 11.0$
$\beta_{AB}-\beta_C$	$-15.0 \leq L \leq 7.9$
$\beta_{AA}-\beta_B$	$64.4 \leq L \leq 94.3$
$\beta_{AA}-\beta_C$	$62.8 \leq L \leq 91.1$
$\beta_{AA}-\beta_{AB}$	$66.0 \leq L \leq 95.0$
$\beta_B-\beta_C$	$-14.5 \leq L \leq 9.9$

TABLE XI. CONFIDENCE INTERVALS FOR CONTRAST, TOTAL SCORES. SCHOOL A SEPARATED INTO TWO CLUSTERS.

The scatter diagram for school A is shown in Figure 1. The regression equations and correlations for the school as a whole and separated into clusters are shown on the figure. The correlation of the pupils in cluster B increased to 0.823, while the correlation of the pupils in cluster A decreased to 0.124.

FIGURE I. SCATTER DIAGRAM AND REGRESSION ANALYSIS, SCHOOL A.



SCHOOL A ———	CLUSTER A — - —	CLUSTER B - - - - - -
$Y = 18.27 + 1.15X$	$Y = 378.8 + 0.097X$	$Y = 129.48 + 0.659X$
$S_Y = 50.84$	$S_Y = 19.43$	$S_Y = 30.43$
$S_{Y \mid X} = 42.62$	$S_{Y \mid X} = 19.28$	$S_{Y \mid X} = 17.29$
$r = 0.545$	$r = 0.124$	$r = 0.823$

VIII. CONCLUSIONS

In the Introduction it was questioned whether a proposed change in a mathematics program would lead to increased math achievement. The author contends that thru statistical analysis this thesis established the increased achievement beyond a reasonable doubt.

From the confidence intervals in Table XI, it is clear that no significant differences existed between cluster B of school A and schools B and C; the confidence intervals for the appropriate contrasts all contain zero. This result, along with the information that cluster B at school A had a sample standard deviation of 30.4 on the posttest score, led the author to conclude that no one was hurt by the program. It afforded the slow achievers the opportunity to receive concentrated instruction at their level. The fact that the concentrated instruction at level V assisted those pupils to advance faster was reflected by the low variability of cluster B's posttest scores compared with schools B and C.

As far as the author can ascertain from the analysis, the average student was unaffected by the program. It is readily apparent that those pupils in levels I and II received the greatest benefit. The means for levels I and II, on the posttest score, was above the 99th percentile of the nation based upon the standardization data of the CAT. On the average, cluster A missed less than two questions

per child out of 117 on the posttest. This spectacular result further suggests that cluster A could have scored even higher on an expanded test. Since the CAT was designed to be used through the fourth grade, the second graders are to be commended on their accomplishment.

It may be questioned whether or not the teachers at the experimental school were concentrating their instruction in the area of the CAT. The author talked to one of the teachers in the experimental school and to the principals of all three schools. He was assured that "teaching to the test" was not done at any of the schools.

It is noted, that, since the experimental teaching program was conceived and implemented by school A teachers, the so called "Hawthorne effect" [Ref. 17] may have been operative. The results are quite striking, however, and the author feels that if such a program were administered by other teachers the improvement indicated here would not be diminished by any large degree. Finally, it is noted that none of the students at any of the schools were aware of the experiment; only the teachers.

It is hoped that this thesis is of some aid in keeping the program at school A; including the extension of it into the other elementary grades. It is further hoped that it can be used as a basis to implement the experimental program at other schools with the school district.

APPENDIX A: SCHOOL CHARACTERISTICS QUESTIONNAIRE

1. How many students are enrolled in this school at each of the following grade levels:

K____	7____
1____	8____
2____	9____
3____	10____
4____	11____
5____	12____
6____	

2. About what percentage of the students who attended this school last year are no longer attending this school (do not count those who have moved because of graduation or are being bussed to other schools)?

______%

3. How old is the main classroom building of this school plant?

______ years old.

4. About what percent of the families of students at this school are represented at a typical meeting of the PTA or similar parent group?

______%

5. Which of the following categories best describe the neighborhood served by this school?

____ a. rural
____ b. residential suburb
____ c. industrial suburb
____ d. small town (5,000 or less)
____ e. city of 5,000 to 50,000
____ f. residential area of a large city (50,000+)
____ g. inner part of a large city (50,000+)

6. About what percentage of students in this school have mothers who are employed outside of the home?

______%

7. From which of the following groups (check all that apply) is formal approval required to initiate new education programs in this school (e.g., team teaching, new curricula, ungraded classrooms, resource rooms, etc.)?

____ Board of Education
____ Superintendent
____ District administration other than Superintendent
____ Parents
____ Teachers
____ No formal approval needed

8. About how long does it usually take to implement a new educational program in this school (i.e., from the time the decision is made to adopt it until the time it is actually introduced)?

______ months

9. (a) (Elementary Schools) What is the copyright date of the regular class reading book used in the third grade at this school? __________

(b) (Junior and senior high schools) What is the copyright date of the regular American history text used in this school? ______

10. About what percentage of the students in this school are white?

______ %

11. What is the annual salary of the principal of this school?

$______

12. What is the starting annual salary of a fully certified beginning teacher in this school system?

$______

13. (Elementary schools only) About what percentage of the students now in Grade 1 in this school attended Kindergarten or its equivalent?

______%

14. About what percentage of the students in this school are living in homes in which there is only one parent?

______%

15. About what percentage of the students in this school speak a langugage other than English outside of school or come from homes in which a language other than English is spoken most of the time?

______%

16. About what percentage of the pupils served by this school fall into each of the categories listed in the chart below (the total should equal 100%)?

Occupational Category	%
children of professionals and managers (doctors, lawyers, engineers, executives, etc.)	
children of white collar workers other than those in (a) above (proprietors, salesman, clerks, etc.)	
children of skilled workers (electricians, carpenters, repair men, factory workers, etc)	
children of unskilled workers (laborers, janitors, dishwashers, etc.)	
TOTAL	100%

17. About how many catalogued volumes are there in the library of this school?

______ volumes

18. What is the average full-time teaching experience of the teaching staff of this school (consider counseling as teaching experience)?

______ years

19. What is the approximate average annual salary of the teaching staff in this school?

$______

20. Please estimate in the chart below the number of hours per week that each of the specified kinds of people are working in this school:

Type of person	# hours per week
Guidance Counselor	
Psychologist	
Child Welfare and Attendance Office	
Nurse	
Speech Therapist	
Remedial Reading Specialist	
English-Second-Language Specialist	

hours per week

Art Teacher
Sex Education Consultant
Librarian
Teacher Aides

APPENDIX B: <u>SCHOOL DISTRICT MINIMUM MATH PROGRAM - GRADE 2</u>

Students in the second grade

-will be able to read numerals through 200 and write numerals through 100, and to count to 100 by 1's and 10's, and to 20 by 5's.

-will gain a working knowledge of the concepts of numbers 0 through 99 and be able to identify the number before and after a given number through 99. They will have some experiences with numbers 100 through 999 including the identification of place value through the hundreds place.

-will be exposed to number words through "ten."

-will gain a working knowledge of the ordinal numerals first through seventh (using teacher-made materials).

-will be able to demonstrate an understanding that there are many ways to write the numeral for a given number.

-will master the addition and subtraction combinations through 15, with emphasis on vertical notation, and will be able to complete the addition and subtraction sentences within these number families.

-will be able to demonstrate an understanding of and be able to add and subtract two-digit numbers which do not require carrying or borrowing.

-will be exposed to and have some practice with addition of three-digit numbers with no carrying.

-will be exposed to and have some practice with addition problems involving carrying and with subtraction problems involving borrowing.

-will be able to demonstrate an understanding of, and be able to work addition problems with, three addends up to and including two-digit numbers.

-will be able to demonstrate an understanding of the meaning of the mathematical vocabulary and symbols included in the minimum program for grade two.

-will be able to demonstrate an understanding of and be able to work oral story problems using the concepts and skills mastered.

-will be able to demonstrate an understanding of the concepts of 1/2 and 1/4 and be exposed to 1/3 as part of a whole.

-will be able to tell time to the nearest hour and half-hour.

-will be able to use pennies, nickels, dimes, quarters, and dollars.

-will have further exposure to a calendar and will develop further understanding of a day, week and month in measuring time.

APPENDIX C: TEST SCORES

SCHOOL A

STUDENT NUMBER	PRETEST CONCEPTS	PRETEST COMPUTATION	PRETEST TOTAL	POSTTEST CONCEPTS	POSTTEST COMPUTATION	POSTTEST TOTAL
1	322	303	290	333	313	310
2	378	293	316	444	373	397
3	346	258	271	333	282	288
4	327	235	250	428	400	405
5	317	260	265	464	400	426
6	304	317	282	464	400	426
7	322	276	280	402	307	322
8	339	280	290	393	309	322
9	395	317	339	428	385	397
10	346	273	287	428	313	331
11	339	286	293	428	385	397
12	395	317	339	464	373	405
13	365	293	310	402	334	348
14	339	293	297	444	348	375
15	309	286	278	384	322	331
16	365	286	305	402	313	327
17	354	263	278	384	311	322
18	292	268	264	444	400	415
19	395	293	323	488	385	426
20	365	293	310	488	400	437
21	346	286	297	384	319	329
22	339	268	280	364	307	315
23	365	293	310	428	400	405
24	354	317	316	402	373	375
25	354	317	316	464	385	415
26	263	254	244	328	288	291
27	339	303	301	464	400	426
28	300	249	252	276	299	281
29	339	235	252	338	278	286
30	296	258	257	348	299	305
31	296	268	265	353	256	273
32	309	280	276	377	326	333
33	365	260	276	344	326	348
34	283	263	257	343	311	312
35	395	317	339	464	400	426
36	332	303	297	402	338	352
37	275	251	246	393	267	291
38	304	280	274	348	286	296
39	327	276	282	358	297	306
40	309	303	282	414	301	319
41	263	249	240	353	258	275
42	296	280	271	358	303	310
43	292	258	256	324	288	289
44	292	303	274	319	280	281

45	322	263	269	428	400	405
46	327	280	284	370	271	289
47	365	286	305	488	400	437
48	378	293	316	488	400	437
49	317	280	280	444	400	415
50	292	268	264	402	385	382
51	327	273	280	464	400	426
52	365	280	301	464	400	426
53	296	276	269	370	334	337
54	313	280	278	393	317	329
55	296	268	265	428	400	405
56	346	260	273	464	363	397
57	339	266	278	428	363	382
58	327	276	282	377	322	329
59	317	273	276	444	400	415
60	300	260	260	428	400	405
61	339	280	290	428	400	405
62	279	268	260	338	301	303
63	328	270	246	343	307	309
64	300	280	273	428	311	329
65	263	249	240	310	284	281
66	271	270	259	314	299	294
67	271	260	251	319	284	284
68	300	276	271	364	313	319
69	296	293	274	377	292	308
70	322	263	269	384	301	315
71	292	286	271	377	315	324
72	271	239	235	343	292	329
73	296	268	265	358	299	308
74	292	249	250	348	264	278
75	304	253	256	281	290	276
76	327	251	261	377	317	325
77	327	303	293	358	317	321
78	339	258	269	348	288	297
79	339	280	290	370	363	352
80	365	233	255	370	296	288
81	332	303	297	414	319	335
82	395	303	330	402	317	331
83	395	261	282	393	385	375
84	365	317	323	414	315	331
85	317	270	274	296	303	291
86	322	286	284	353	260	276
87	346	286	297	393	363	364
88	292	248	248	393	311	324
89	322	261	268	348	299	305
90	332	286	290	377	324	331
91	322	265	271	353	262	278
92	395	317	339	428	400	405
93	395	268	293	343	309	310
94	322	286	284	364	269	286

SCHOOL B

1	365	317	323	414	385	389
2	339	293	297	393	305	319
3	296	258	257	338	251	265
4	162	233	193	328	262	270
5	378	317	330	464	354	389
6	365	317	323	414	373	382
7	346	303	305	377	338	342
8	395	303	330	464	343	376
9	354	303	310	377	354	352
10	365	303	316	464	328	355
11	317	276	278	364	315	321
12	339	303	301	377	286	303
13	332	317	301	488	348	389
14	354	317	316	428	385	405
15	339	286	293	393	348	355
16	346	286	297	370	315	322
17	300	270	268	353	275	288
18	327	293	290	414	324	339
19	254	265	250	310	290	286
20	292	253	252	300	286	280
21	339	235	252	348	311	313
22	332	266	276	428	354	375
23	346	286	297	414	334	352
24	287	242	243	328	292	294
25	309	246	252	296	262	258
26	309	273	273	370	284	300
27	278	237	259	402	292	312
28	313	261	265	348	269	281
29	296	254	255	338	305	306
30	279	260	254	319	277	278
31	300	251	254	343	307	309
32	339	268	280	338	290	296
33	322	263	269	393	267	291
34	300	261	261	353	269	283
35	296	233	239	291	269	262
36	339	317	305	384	324	333
37	317	293	284	353	315	318
38	271	260	251	300	251	252
39	292	263	260	358	301	309
40	354	276	293	358	309	315
41	304	265	265	343	303	306
42	327	273	280	364	305	313
43	327	276	282	314	277	276
44	300	228	237	328	264	271
45	339	276	287	328	299	299
46	327	260	268	364	311	318
47	296	249	251	343	260	273
48	296	256	256	228	256	265
49	309	265	267	338	258	270

SCHOOL C

1	292	242	244	343	278	288
2	378	251	269	384	313	324
3	365	254	271	393	313	325
4	296	270	267	338	262	273
5	309	235	274	319	313	306
6	339	286	293	338	264	275
7	279	246	243	324	275	278
8	313	254	260	370	311	319
9	378	303	323	377	331	337
10	304	246	251	333	284	289
11	332	293	293	358	331	331
12	322	266	273	343	313	313
13	395	317	339	414	385	389
14	378	256	274	402	326	339
15	309	235	244	333	313	310
16	275	248	243	314	260	263
17	292	239	242	296	288	280
18	327	317	297	364	309	316
19	263	231	226	314	275	275
20	259	239	230	278	247	240
21	395	303	330	402	338	352
22	327	254	264	314	322	310
23	378	317	330	353	317	319
24	378	249	268	370	307	316
25	332	293	293	377	348	348
26	313	251	257	358	267	283
27	395	303	330	464	363	397
28	378	280	305	402	334	348
29	354	249	265	338	331	324
30	322	293	287	393	334	345
31	287	253	251	348	338	331
32	346	303	305	393	242	310
33	317	270	274	384	326	335
34	292	286	271	377	311	321
35	339	293	297	384	343	348
36	271	249	243	358	249	270
37	354	286	301	393	331	342
38	317	270	274	370	328	333
39	317	303	287	348	317	318
40	346	317	310	358	324	325
41	313	273	274	358	292	303
42	309	303	282	364	324	327
43	309	256	259	348	319	319
44	313	280	278	338	284	291
45	378	303	323	444	290	315
46	309	265	267	348	284	294
47	313	276	276	348	331	327
48	395	317	339	488	348	389
49	263	265	252	305	251	255
50	327	266	274	343	301	305

51	354	317	316	377	338	342
52	317	258	264	370	328	333
53	300	270	268	328	297	297
54	378	303	323	377	334	339
55	317	303	287	393	328	339
56	254	190	205	255	284	263
57	283	244	243	264	307	315
58	346	317	310	370	286	302
59	354	293	305	402	385	382
60	354	254	269	353	313	316
61	395	317	339	488	385	426
62	309	303	282	343	297	302
63	279	263	256	271	277	262
64	346	303	305	393	348	355

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13. ABSTRACT

The mathematics program of an elementary school was changed so that the classes were partitioned into smaller sections of comparable ability. This change reduced the pupil-teacher ratio and the length of the instruction period. The proposal produced an unanswered question: Will there be an increased achievement gain during the school year which can be attributed to the program change? Two schools served as control groups for the experiment. The California Achievement Test, 1970 Edition, was used in a pretest-posttest design to measure achievement levels before and after the elapsed time of the experiment. The question was answered in the affirmative through the use of various statistical techniques: randomized matched subjects design, analysis of covariance and the non-parametric Mann-Whitney U test. The several techniques were applied since no single standard practice was available for this problem.

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KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Measuring and Evaluating Change						
School Characteristics Questionnaire						
Pretest-Posttest Design						
California Achievement Test						
Randomized Matched Subject Design						
Analysis of Variance						
Mann-Whitney U test						
Cluster Analysis Computer Program						

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c.1 An experiment in partitioning second-grade mathematics classes into smaller sections of comparable ability.